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一类时滞非自治 Lotka – Volterra 扩散生态系统的全局吸引性*

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摘 要： 研究了一类 n 维纯时滞 Lotka – Volterra 扩散生态系统. 通过一种新的构造 Lyapunov 泛函的方法，获得了一些全局渐近稳定的新的结果，推广和改进了某些已知结果，具有现实指导意义。

关键词： 时滞；全局吸引性；非自治；扩散
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文[1]提出如下的时滞非自治捕食系统：

$$\begin{cases} \dot{x}_1 = x_1[r_1 - a_1x_1 - b_1x_1(t-\tau_1(t)) - \\ \qquad \int_{-\infty}^{0} k_1(t,s)x_1(t+s)\mathrm{d}s - \dfrac{cy}{1+\alpha x_1}] + \\ \qquad \sum\limits_{i=2}^{n} D_{i1}(x_i - x_1), \\ \dot{x}_i = x_i[r_i - a_ix_i - b_ix_i(t-\tau_i(t)) - \\ \qquad \int_{-\infty}^{0} k_i(t,s)x_i(t+s)\mathrm{d}s] + \\ \qquad \sum\limits_{j=1}^{n} D_{ji}(x_j - x_i), (i = 2,3,\cdots,n), \\ \dot{y} = y[-r_{n+1} - a_{n+1}y - b_{n+1}y(t-\tau_{n+1}(t)) - \\ \qquad \int_{-\infty}^{0} k_{n+1}(t,s)y(t+s)\mathrm{d}s + \dfrac{ex_1}{1+\alpha x_1}]. \end{cases} \tag{1}$$

其中 $x_i(t)(i=1,2,\cdots,n)$ 分别表示 t 时刻斑块 i 中种群的密度，$D_{ij}(t)(D_{ii}(t)=0)(i,j=1,2,\cdots,n)$ 表示种群 x 从斑块 j 到斑块 i 的扩散系数。更多的有关系统（1）的生态学背景可参见文［1 – 9］. 文［1］利用微分方程理论和构造适当的 Lyapunov 函数得到了生物系统（1）的持续生存和全局吸引性. 然而文［1］在证明持续生存和全局吸引性是在假设 $a_i(t)+b_i(t)>0, i=1,2,\cdots,n$ 的条件下证得的（对这类系统我们称为非纯时滞系统），当（1）为纯时滞系统（即各项均为时滞），则文［1］证明的动力学行为方法失效。Wang 等[3]指出在实际的生态系统中，可能纯时滞生态系统是更为符合实际的生态系统。因此在假设 $a_i(t)=0,(i=1,2,\cdots,n+1)$ 即纯时滞系统的条件下，证明生物系统（1）的持续生存和全局吸引性就显得有意义和尤为必要。本文提出如下纯时滞非自治 n 种群捕食模型：

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1(t) - a_1(t)x_1(t-\tau_1(t)) - \\ \qquad \int_{-\infty}^{0} k_1(t,s)x_1(t+s)\mathrm{d}s - \dfrac{c(t)y(t)}{1+\alpha(t)x_1(t)}] + \\ \qquad \sum\limits_{i=2}^{n} D_{i1}(t)(x_i(t) - x_1(t)), \\ \dot{x}_i(t) = x_i(t)[r_i(t) - a_i(t)x_i(t-\tau_{n+1}(t)) - \\ \qquad \int_{-\infty}^{0} k_i(t,s)x_i(t+s)\mathrm{d}s] + \\ \qquad \sum\limits_{j=1}^{n} D_{ji}(t)(x_j(t) - x_i(t)), (i = 2,3,\cdots,n), \\ \dot{y}(t) = y(t)[-r_{n+1}(t) - a_{n+1}(t)y(t-\tau_{n+1}(t)) - \\ \qquad \int_{-\infty}^{0} k_{n+1}(t,s)y(t+s)\mathrm{d}s + \dfrac{e(t)x_1(t)}{1+\alpha(t)x_1(t)}] \end{cases} \tag{2}$$

通过改进文［1 – 5，7］的方法，本文将给出系统（2）持续生存和全局吸引性充分条件。假设

（H1）$a_i(t)$，$r_i(t)$，$c(t)$，$e(t)$，$\alpha(t)$ 和 $D_{ij}(t)$ $(D_{ii}(t)=0)(i,j=1,2,\cdots,n,n+1)$ 都是 $[0,+\infty)$ 上 正的连续有界函数；

（H2）$k_i(t,s)>0, \int_{-\infty}^{0} k_i(t,s)\mathrm{d}s, (i=1,2,\cdots,n+1)$ 关于 $t\in\mathbf{R}$ 一致连续有界；

（H3）$\tau_i(t),(i=1,2,\cdots,n+1)$ 是 $[0,+\infty)$ 上的非负、有界、连续可微函数且 $\inf\limits_{t>0}(1-\dot{\tau}(t))>0$。令 $\tau^* = \sup\limits_{t\geq 0}\{\tau_i(t)\}$，$\tau_* = \inf\limits_{t\geq 0}\{\tau_i(t)\}$，$(i=1,2,\cdots,n+1)$，于是 $0\leq\tau_*<\tau^*<+\infty$. 令 $\sigma_i(t)=t-\tau_i(t)$，则 $\sigma_i^{-1}(t)$ 是 $\sigma_i(t)$ 的反函数。

为方便起见，我们介绍下面的符号及定义：

$$C_h = \left\{\phi \in C(\mathbf{R}_-,\mathbf{R}^{n+1}) : \int_{-\infty}^{0} \sup_{s\leq t\leq 0} |\phi(t)|\,\mathrm{d}s < \infty\right\},$$

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令 $C_h^+ = \{\phi = (\phi_1,\phi_2,\cdots,\phi_{n+1}) \in C_h:\phi_i(0) > 0(i = 1,2,\cdots,n+1)\}$。根据种群动力学的实际情况，我们总是假设系统（2）的解满足如下条件：

$$x_i(\theta) = \phi_i(\theta),y(\theta) = \phi_{n+1}(\theta),$$
$$\theta \in (-\infty,0],(i = 1,2,\cdots,n) \tag{3}$$

这里 $\phi=(\phi_1,\phi_2,\cdots,\phi_{n+1}) \in C_h^+$。因此，如果选择 C_h^+ 作为系统（2）的初值函数空间，根据时滞泛函微分方程的基本原理[10]，对于任意的 $\phi=(\phi_1,\phi_2,\cdots,\phi_{n+1}) \in C_h^+$，$\phi(0)>0$ 系统（2）存在满足初值条件（3）唯一的正解 $X(t,\phi)$。因此，文章以下部分总是假设 ϕ 满足 $\phi \in C_h^+$，$\phi(0)>0$。

引进记号 $f^l=\inf\limits_{t>0} f(t)$，$f^m=\sup\limits_{t>0} f(t)$，其中 $f(t)$ 是 $[0,+\infty)$ 上的连续有界函数。

1　全局吸引性

现在通过发展文［1，4，5，7］的构造 Lyapunov 泛函的技巧构造适当的 Lyapunove 泛函，讨论系统（2）的正解的全局吸引性.

引理 1　假设条件(H1)－(H3)成立，且满足如下条件：

(H4) $r_{n+1}^l < \dfrac{e^m M_1}{1+\alpha^l M_1}$;

(H5) $r_1^l > p_1 + c^m M_{n+1} + q_1$，$r_{n+1}^m < h - p_{n+1}$，$r_i^l > p_i + q_i,(i = 2,3,\cdots,n)$;

则系统（2）满足初始条件（3）的任意解 $X(t) = (x_1(t),x_2(t),\cdots,x_n(t),y(t))$，存在 $T>0$，使得当 $t>T$ 时有：

$$m_i \le x_i(t) \le M_i(i=1,2,\cdots,n)$$
$$m_{n+1} \le y(t) \le M_{n+1} \tag{4}$$

其中

$$m_1^* = \frac{r_1^l - p_1 - q_1}{a_1^m}\exp\{(r_1^l - p_1 - c^m M_{n+1} - q_1)\tau_*\},$$

$$m_i^* = \frac{r_i^l - p_i - q_i}{a_i^m}\exp(r_i^l - p_i - q_i)\tau_*),(i = 2,\cdots,n),$$

$$m_{n+1}^* = (-r_{n+1}^M - p_{n+1} + h)/a_{n+1}^m\exp\{(-r_{n+1}^M - p_{n+1} + h)\tau_*\},$$

$$M_i^* = \frac{r_i^m}{a_i^l}\exp\{r_i^m\tau^*\}(i = 1,2,\cdots,n),$$

$$M_{n+1}^* = \frac{e^m M_1/(1+\alpha^l M_1) - r_{n+1}^l}{a_{n+1}^l}\exp\{(e^m M_1/(1+\alpha^l M_1) - r_{n+1}^l)\tau^*\},p_i = M_i\int_{-\infty}^0 k_i^m(s)\mathrm{d}s,$$

$q_i = \sum\limits_{j=2}^n D_{ji}^m,h = e^l m_1/(1+\alpha^m m_1)$. $m_i,M_i,(i = 1,2,\cdots,n+1)$ 是适当选取的满足 $0<m_i<M_i^*$，$M_i>0$ 的常数。

证明　见文［1］定理2.1。

引理 2　设 $X(t) = (x_1(t),x_2(t),\cdots,x_n(t),y(t))$ 和 $U(t) = (u_1(t),u_2(t),\cdots,u_n(t),v(t))$ 是系统（2）的任意两个正解。则当 $t\ge T$ 时

$$\operatorname{sgn}(x_i(t) - u_i(t))\sum_{j=1}^n D_{ij}(t)\frac{x_j(t)u_i(t) - x_i(t)u_j(t)}{x_i(t)u_i(t)}$$
$$\le \sum_{j=1}^n D_{ij}(t)\frac{1}{m}|x_j(t) - u_j(t)| \tag{5}$$

其中 $m=\min\{m_i,\ i=1,2,\cdots,n\}$。

定理 1　如果系统（2）满足条件(H1)－(H5)且存在常数 $c_i(i=1,2,\cdots,n+1)$ 使得

(H6) $\lim\limits_{t\to\infty}\inf A_i(t) > 0(i = 1,2,\cdots,n+1)$

成立，其中

$$A_1(t) = c_1(a_1(t) - [1 + M_1 o_1(t)]c(t)\alpha(t)k^2(t)M_{n+1} - g_1(t) - c(t)o_1(t)k(t)M_{n+1} - h_1(t) - f_1(t)(1 + M_1 o_1(t))) - c_{n+1}(1 + o_{n+1}(t))\cdot e(t)k^2(t) - \frac{1}{m}\sum_{j=2}^n c_j D_{1j}(t)$$

$$A_i(t) = c_i(a_i(t) - g_i(t) - h_i - f_i(t)(1 + M_i o_i(t))) - \frac{1}{m}\sum_{j=1}^n c_j D_{ji}(t),\quad i = 2,\cdots,n$$

$$A_{n+1}(t) = c_{n+1}(a_{n+1}(t) - M_{n+1}f_{n+1}(t)o_{n+1}(t) - g_{n+1}(t) - \frac{c(t)o_{n+1}(t)M_1}{1+\alpha(t)M_1} - f_{n+1}(t) - \frac{M_{n+1}}{M_1}h_1 - c_1(1 + M_1 o_1(t))c(t)k(t).$$

其中 $g_i(t) = \int_t^{\sigma_i^{-1}(t)} a_i(s)\mathrm{d}s[r_i(t) + [a_i(t) + \int_{-\infty}^0 k_i(t,s)\mathrm{d}s]M_i$，$o_i(t) = \int_t^{\sigma_i^{-1}(t)} a_i(s)\mathrm{d}s]$，

$$h_i(t) = M_i\int_{\sigma_i^{-1}(t)}^{\sigma_i^{-1}(\sigma_i^{-1}(t))} a_i(\theta)\mathrm{d}\theta\frac{a_i(\sigma_i^{-1}(t))}{1 - \dot{\tau}_i(\sigma_i^{-1}(t))},$$

$f_i(t) = \int_{-\infty}^0 k_i(t-s,s)\mathrm{d}s,k(t)=1/(1+\alpha(t)m_1)$。则系统（2）的任意正解是全局渐近稳定的。

证明　设 $X(t) = (x_1(t),x_2(t),\cdots,x_n(t),y(t))$ 和 $U(t) = (u_1(t),u_2(t),\cdots,u_n(t),v(t))$ 是系统（2）的任意两个正解。下面我们逐一考虑 Lyapunove 泛函：令 $V_{11} = |\ln x_1(t) - \ln u_1(t)|$，

计算 $V_{11}(t)$ 沿着系统（2）的解的上右导数，对所有的 $t>T+\tau^*$，由（4），（5）有

$$D^+V_{11}(t) = \left(\frac{\dot{x}_1(t)}{x_1(t)} - \frac{\dot{u}_1(t)}{u_1(t)}\right)\operatorname{sgn}(x_1(t) - u_1(t)) = \operatorname{sgn}(x_1(t) - u_1(t))\{-a_1(t)(x_1(t) - u_1(t)) + a_1(t)\int_{-\tau_1(t)}^t\{x_1(u)[r_1(u) - a_1(u)x_1(u-\tau_1(u)) - \int_{-\infty}^0 k_1(u,s)x_1(u+s)\mathrm{d}s - \frac{c(u)y(u)}{1+\alpha(u)x_1(u)}] -$$

$u_1(u)[r_1(u)-a_1(u)u_1(u-\tau_1(u))-$
$\int_{-\infty}^{0}k_1(u,s)u_1(u+s)\mathrm{d}s-\frac{c(u)v(u)}{1+\alpha(u)u_1(u)}]\}\mathrm{d}u-$
$\int_{-\infty}^{0}k_1(t,s)(x_1(t+s)-u_1(t+s))\mathrm{d}s-$
$(\frac{c(t)y(t)}{1+\alpha(t)x_1(t)}-\frac{c(t)v(t)}{1+\alpha(t)u_1(t)})+$
$\sum_{i=2}^{n}D_{i1}(t)\frac{x_i(t)u_1(t)-u_i(t)x_1(t)}{x_1(t)u_1(t)}\}\leqslant$
$-a_{11}(t)\mid x_1(t)-u_1(t)\mid+a_{11}(t)\int_{-\tau_1(t)}^{t}\{\mid x_1(u)-$
$u_1(u)[r_1(u)-a_1(u)x_1(u-\tau_1(u))-$
$\int_{-\infty}^{0}k_1(u,s)x_1(u+s)\mathrm{d}s-\frac{c(u)y(u)}{1+\alpha(u)x_1(u)}]+$
$u_1(u)[a_1(u)\mid x_1(u-\tau_1(u))-u_1(u-\tau_1(u))\mid+$
$\int_{-\infty}^{0}k_1(u,s)(x_1(u+s)-u_1(u+s))\mathrm{d}s+$
$(\frac{c(u)y(u)}{1+\alpha(u)x_1(u)}-\frac{c(u)v(u)}{1+\alpha(u)u_1(u)})]\}\mathrm{d}u+$
$\int_{-\infty}^{0}k_1(t,s)\mid(x_1(t+s)-u_1(t+s))\mid\mathrm{d}s+$
$c(t)k(t)\mid y(t)-v(t)\mid+c(t)\alpha(t)k^2(t)M_{n+1}\cdot$
$\mid x_1(t)-u_1(t)\mid+\frac{1}{m}\sum_{i=2}^{n}D_{i1}(t)\mid x_i(t)-u_i(t)\mid \quad (6)$

定义

$V_{12}(t)=\int_{t}^{\sigma_1^{-1}(t)}\int_{\sigma_1(s)}^{t}a_1(s)[r_1(u)-a_1(u)\cdot$
$x_1(u-\tau_1(u))+\int_{-\infty}^{0}k_1(u,s)(x_1(u+s)-$
$u_1(u+s))\mathrm{d}s+\int_{-\infty}^{0}\int_{t+s}^{t}k_1(u-s,s)\mid x_1(u)-$
$u_1(u)\mid\mathrm{d}u\mathrm{d}s+\int_{t}^{\sigma_1^{-1}(t)}\int_{\sigma_1(s)}^{t}a_1(s)u_1(u)\cdot$
$[\mid x_1(u-\tau_1(u))-u_1(u-\tau_1(u))\mid+$
$(\frac{c(u)y(u)}{1+\alpha(u)x_1(u)}-\frac{c(u)v(u)}{1+\alpha(u)u_1(u)})]\}\mathrm{d}u-$
$\int_{\infty}k_1(u,s)x_1(u+s)ds-\frac{c(u)y(u)}{1+\alpha(u)x_1(u)}]$
$\mid x_1(u)-u_1(u)\mid\mathrm{d}u\mathrm{d}s \quad (7)$

由（6）和（7）式，当 $t>T+\tau^*$ 时

$D^+V_{11}(t)+V_{12}(t)\leqslant-a_1(t)\mid x_1(t)-u_1(t)\mid+$
$c(t)k(t)\mid y(t)-v(t)\mid+c(t)\alpha(t)M_{n+1}k(t)\cdot$
$\mid x_1(t)-u_1(t)\mid+[g_1(t)+c(t)\alpha(t)k(t)o_1(t)M_{n+1}]\cdot$
$\mid x_1(t)-u_1(t)\mid+M_1o_1(t)[\mid x_1(t-\tau_1(t))-$
$u_1(t-\tau_1(t))\mid+\frac{1}{m}\sum_{i=2}^{n}D_{i1}(t)\mid x_i(t)-u_i(t)\mid+$
$\int_{-\infty}^{0}k_1(t,s)\mid x_1(t+s)-u_1(t+s)\mid\mathrm{d}s+$
$c(t)k(t)\mid y(t)-v(t)\mid+c(t)\alpha(t)k^2(t)M_{n+1}\mid x_1(t)-$
$u_1(t)\mid]+f_1(t)\mid x_1(t)-u_1(t)\mid \quad (8)$

现在定义：$V_1(t)=V_{11}(t)+V_{12}(t)+V_{13}(t) \quad (9)$

其中 $V_{13}=M_1\int_{t-\tau_1(t)}^{t}\int_{\sigma_1^{-1}(l)}^{\sigma_1^{-1}}a_1(\theta)\frac{a_1(\sigma_1^{-1}(l))}{1-\dot{\tau}_1(\sigma_1^{-1}(l))}\cdot$
$\mid x_1(l)-u_1(l)\mid\mathrm{d}\theta\mathrm{d}l+M_1\int_{t}^{\sigma_1^{-1}(t)}a_1(s)\mathrm{d}s\cdot$
$\int_{-\infty}^{0}\int_{t+s}^{t}k_1(\theta-s,s)\mid x_1(\theta)-u_1(\theta)\mid\mathrm{d}\theta\mathrm{d}s.$

则由（8），（9）式知：当 $t>T+\tau^*$ 时，有

$D^+V_1(t)\leqslant-(a_1(t)-[1+M_1o_1(t)]c(t)\alpha(t)k^2(t)M_{n+1}-$
$g_1(t)-c(t)o_1(t)k(t)M_{n+1}-f_1(t)-$
$h_1-M_1o_1(t)f_1(t))\mid x_1(t)-u_1(t)\mid+$
$(1+M_1o_1(t))c(t)k(t)\mid y(t)-v(t)\mid+$
$\frac{1}{m}\sum_{j=1}^{n}D_{ij}(t)\mid x_j(t)-u_j(t)\mid \quad (10)$

类似的可定义 $V_i(t)=\sum_{j=1}^{3}V_{i,j}(t)$，$i=2,3,\cdots,n$。当 $t>T+\tau^*$ 时，

$$D^+V_i(t)\leqslant-[a_i(t)-g_i(t)-f_i(t)-h_i(t)-M_io_i(t)f_i(t)]\mid x_i(t)-u_i(t)\mid+\frac{1}{m}\sum_{j=1}^{n}D_{ij}(t)\mid x_j(t)-u_j(t)\mid \quad (11)$$

同样的我们可定义 $V_{n+1}(t)=\sum_{j=1}^{3}V_{n+1,j}(t)$，当 $t>T+\tau^*$ 时，

$D^+V_{n+1}(t)\leq-(a_{n+1}(t)-g_{n+1}(t)+\frac{c(t)M_1}{1+\alpha(t)M_1}\cdot$
$\int_{t}^{\sigma_{n+1}^{-1}(t)}a_{n+1}(s)\mathrm{d}s-o_{n+1}(t)-h_1(t)M_{n+1}/M_1-$
$f_{n+1}(t))\mid y(t)-v(t)\mid-(1+M_{n+1}\cdot$
$\int_{t}^{\sigma_{n+1}^{-1}(t)}a_{n+1}(s)\mathrm{d}s)e(t)k^2(t)\mid x_1(t)-u_1(t)\mid \quad (12)$

现在我们构造 Lyapunov 泛函如下：$V(t)=\sum_{i=1}^{n+1}c_iV_i$

由（10），（11）和（12）式知：当 $t\geq T+\tau^*$ 时有

$$D^+V(t)\leq-\sum_{i=1}^{n}A_i(t)\mid x_i(t)-u_i(t)\mid-A_{n+1}(t)\mid y(t)-v(t)\mid$$

其中 A_i 为定理1中所定义。

由条件（H6）知：存在 $\delta>0$，T^* 使得当 $t>T^*>T+\tau^*$ 时

$$D^+V(t)\leq-\sum_{i=1}^{n}A_I(t)\mid x_i(t)-u_i(t)\mid-A_{n+1}(t)\cdot\mid y(t)-v(t)\mid\leq-\delta(\sum_{i=1}^{n}\mid x_i(t)-u_i(t)\mid+\mid y(t)-v(t)\mid) \quad (13)$$

对所有的 $t\geq T^*$，并对（13）两边在区间 $[T^*,t]$ 上积分，得 $V(t)+\delta\int_{T^*}^{t}\sum_{i=1}^{n}\mid x_i(s-u_i(s)\mid+\mid y(t)-v(t)\mid\mathrm{d}s\leq V(T^*)$。所以，$V(t)$ 在区间 $[T^*,+\infty)$ 上有界，并且 $\int_{T^*}^{\infty}\mid x_i(s)-u_i(s)\mid\mathrm{d}s<\infty$，$(i=1,2,\cdots,n)$，$\int_{T^*}^{\infty}\mid y(s)-v(s)\mid\mathrm{d}s<\infty$.

根据引理 1，我们知道 $X(t)$ 和 $U(t)$ 在区间 $[T^*, +\infty]$ 上有界。因此 $|x_i(t)-u_i(t)|, i=1, 2, \cdots, n$，在区间 $[T^*, +\infty)$ 上有界并一致连续. 根据文［10］中的引理 1.22，得

$$\lim_{t\to\infty}|x_i(t)-u_i(t)|=0(i=1,2,\cdots,n),$$

$$\lim_{t\to\infty}|y(t)-v(t)|=0。证毕。$$

若 $\tau_i(t)=0, k_i(t,s)=0, (i=1,2,\cdots,n+1)$ 则系统（2）退化为如下的无时滞扩散 n 种群捕食者生物系统

$$\begin{cases}\dot{x}_1(t)=x_1(t)[r_1(t)-a_1(t)x_1(t)-\dfrac{c(t)y(t)}{1+\alpha(t)x_1(t)}]+\\ \qquad\displaystyle\sum_{i=2}^{n}D_{i1}(x_i(t)-x_1(t)),\\ \dot{x}_i(t)=x_1(t)[r_1(t)-a_i(t)x_i(t)+\\ \qquad\displaystyle\sum_{j=1}^{n}D_{ji}(x_j(t)-x_i(t)),(i=2,3,\cdots,n)\\ \dot{y}(t)=y(t)[r_{n+1}(t)-a_{n+1}(t)y(t)+\dfrac{e(t)x_1(t)}{1+\alpha(t)x_1(t)}]\end{cases}\tag{14}$$

对系统（14）有如下结论：

推论 1　假设条件(H1)-(H5)成立，且存在 c_i $(i=1, 2, \cdots, n+1)$，使得

$$\liminf_{t\to\infty}\begin{Bmatrix}c_i(a_1(t)-\alpha(t)c(t)k^2(t)M_{n+1})-\\ c_{n+1}e(t)k^2(t)-\dfrac{1}{m}\displaystyle\sum_{j=2}^{n}c_jD_{j1}(t);\\ c_ia_i-\dfrac{1}{m}\displaystyle\sum_{j=1}^{n}c_jD_{ji}(t),(i=1,2,\cdots,n);\\ c_{n+1}a_{n+1}-c_1c(t)k_t).\end{Bmatrix}$$

存在且大于 0，则对系统（14）的任意两个正解 $X(t)=(x_1(t),x_2(t),\cdots,x_n(t),y(t)), U(t)=(u_1(t),u_2(t),\cdots,u_n(t),v(t))$ 都有：

$$\lim_{t\to\infty}|x_i(t)-u_i(t)|=0\quad(i=1,2,\cdots,n),$$

$$\lim_{t\to\infty}|y(t)-v(t)|=0。$$

作为本文主要结果的应用，考虑如下的时滞扩散捕食者生态系统：

$$\begin{cases}\dot{x}_1(t)=x_1(t)[r_1(t)-a_1(t)x_1(t-\tau_1(t))-\\ \qquad\displaystyle\int_{-\infty}^{0}k_1(t,s)x_1(t+s)\mathrm{d}s-\dfrac{c(t)y(t)}{1+\alpha(t)x_1(t)}]\\ \qquad D_1(t)(x_2(t)-x_1(t)),\\ \dot{x}_2(t)=x_2(t)[r_2(t)-a_2(t)x_2(t-\tau_2(t))-\\ \qquad\displaystyle\int_{-\infty}^{0}k_2(t,s)x_2(t+s)\mathrm{d}s]+\\ \qquad D_2(t)(x_1(t)-x_2(t)),\\ \dot{y}(t)=y(t)[-r_3(t)-a_3(t)y(t-\tau_3(t))-\\ \qquad\displaystyle\int_{-\infty}^{0}k_3(t,s)y(t+s)\mathrm{d}s+\dfrac{e(t)x_1(t)}{1+\alpha(t)x_1(t)}].\end{cases}\tag{15}$$

且满足相应的条件(H1)-(H5)。则有如下结论：

推论 2　如果系统（2）存在常数 c_i $(i=1, 2, 3)$ 满足条件条：$\liminf_{t\to\infty}A_i(t)>0(i=1,2,3)$。其中

$$A_1(t)=c_1(a_1(t)-[1+M_1o_1(t)]\alpha(t)c(t)k^2(t)M_2-g_1(t)-o_1(t)c(t)k(t)M_2-f_1(t)-h_1(t)-M_1o_1(t)f_1(t))-c_2(1+M_2o_2(t))\cdot e(t)k^2(t)-c_2\frac{D_2(t)}{m};$$

$$A_2(t)=c_2(a_2(t)-g_2(t)-M_2o_2(t)f_2(t)-h_2(t)-f_2(t))-\frac{c_1D_1(t)}{m};$$

$$A_3(t)=c_3(a_3(t)-M_3o_3(t)f_3(t)-g_3(t)-o_3(t)\frac{c(t)M_1}{1+\alpha(t)M_1}-f_3(t)-h_1(t)M_2/M_1)-c_1(1+M_1o_1(t))c(t)k(t)$$

则对系统（15）的任意两个正解 $X(t)=(x_1(t), x_2(t),y(t))$ 和 $U(t)=(u_1(t),u_2(t),v(t))$ 都有

$$\lim_{t\to\infty}|x_i(t)-u_i(t)|=0,(i=1,2),$$

$$\lim_{t\to\infty}|y(t)-v(t)|=0。$$

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Globally Asymptotic Stability of a Nonautonomous Lotka – Volterra System with Delays and Dispersion

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Abstract: A class of n-dimensional pure-delayed Lotka-Volterra ecological system with dispersion is investigated. By constructing suitable improved Lyapunov functional, some new results for the global asymptotical stability are obtained, which are basical extensions or improvements of some known results, with realistically instructive significance.

Key words: time delays; globally asymptotic stability; dispersion

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